

LIQUID WATER IN SQUALL LINES AND HURRICANES AT AIR TEMPERATURES LOWER THAN -40°C .¹

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ABSTRACT

In convective clouds jet aircraft encounter icing at air temperatures lower than -60°C . It may be reasoned that this results from (1) the melting of frozen droplets due to dynamic or frictional heating in the quasi-potential flow around the air foil; (2) the impact of precipitation consisting of ice spheres with liquid water cores; (3) the possibility that raindrops are carried upward by vertical currents so rapidly that the water temperature remains higher than the spontaneous nucleation temperature (-40°C .) while the air temperature is much lower; or (4) the possibility that the spontaneous nucleation temperature is pressure dependent or otherwise variable. This paper investigates the third of these possibilities and finds that the difference in temperature of liquid water and air at 12 km. can be no more than a few degrees except in the case of very large drops carried through deep layers of cloud by intense steady-state updrafts of the kind associated with hail-producing thunderstorms.

I. INTRODUCTION

In 1946, Schaefer [7] and his associates demonstrated in laboratory experiments that the degree to which liquid water drops can be supercooled depends upon the kind of particles available for ice nucleation. Some natural dusts and clays were found to be effective at temperatures as high as -8°C .; others were not effective at temperatures higher than -35°C . In clean laboratory air it was found that freezing occurred nevertheless at about -40°C . From these experiments -40°C . has come to be known as the temperature of spontaneous nucleation where freezing occurs in the absence of nucleating particles.

Recently, Schaefer [8] repeated some of these experiments in the free atmosphere at Yellowstone National Park. The regular eruptions of the geyser "Old Faithful" were observed on mornings when surface temperatures were lower than -40°C . and it was reported that in the cold surface air the geyser cloud readily produced ice crystals. However, on those days when the temperature was above -40°C ., the cloud often remained in a liquid state.

Despite these experimental results, much evidence has accumulated during the last decade of aircraft icing at elevations where air temperatures were much lower than -40°C . Also, a number of laboratory experiments have succeeded in cooling water droplets to temperatures below -40°C . (e.g., Weickmann [11]). Some of these results have been questioned on the grounds that insufficient safeguards had been taken to eliminate impurities which might inhibit freezing. Birstein [1], for

example, demonstrated that by introducing small quantities of methyl and ethyl amines into the cloud, water droplets could be cooled below -70°C . without freezing. Aircraft icing, on the other hand, would seem to provide more conservative evidence of the existence of liquid water. Unfortunately, most of the reports of icing have been visual or subjective estimates. In many instances there have been no means of measuring the air temperature accurately.

Only recently have research aircraft recorded information which provides more objective evidence that clear ice may form at air temperatures lower than -60°C .

In the spring season of 1960-62 jet aircraft supporting the National Severe Storms Project (NSSP) in Oklahoma flew through convective towers of squall line systems at elevations between 30,000 and 40,000 ft. investigating the distribution of turbulence in relation to radar echoes [10]. At air temperatures ranging from -45° to -55°C ., these aircraft encountered numerous instances in which clear ice accumulated rapidly on the windshield and various structural members of the aircraft. A number of examples of this kind of icing has also been obtained by research aircraft of the National Hurricane Research Project (NHRP) in flights through hurricanes. Figure 1 shows (a) the accumulation of ice during penetration of the wall cloud, and (b) the character of the ice as it was melting away in the eye of hurricane Esther, September 17, 1961. These are frames from time-lapse photographs made in a W-57 aircraft flying at 45,000 ft. at a true airspeed of approximately 450 kt. The exterior mounting of the camera is shown in figure 2. The ice formed on the flat plate of optical glass, electrically heated, through

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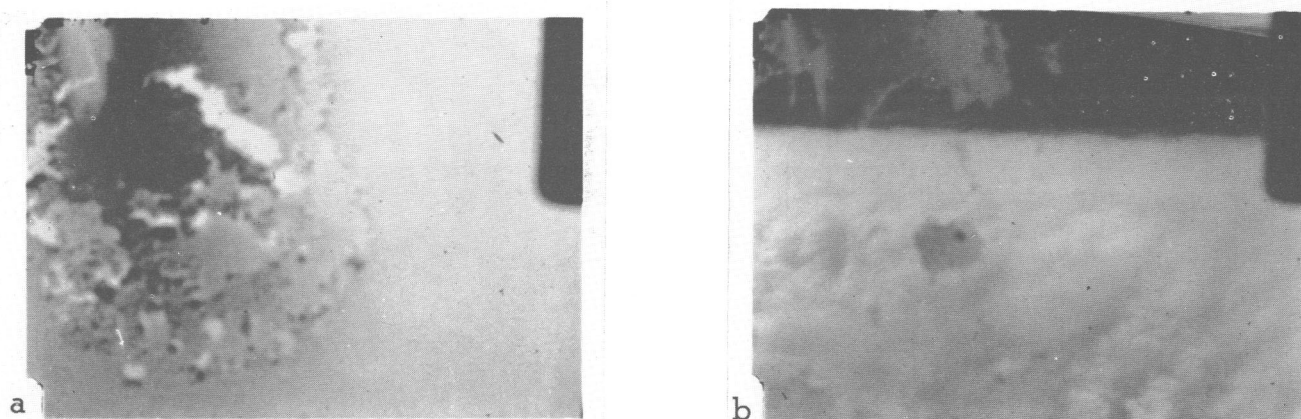


FIGURE 1.—Accumulation of clear icing on an electrically heated pane of optical glass in the housing of the time-lapse camera on a W-57 (Canberra) aircraft (see fig. 2). (a) shows the accumulation during transit of the eye wall of hurricane Esther, September 17, 1961; (b) taken during transit of the clear air of the eye, shows the ice being melted away by the heating element.

which the camera peered. The accumulation occurred in an area where temperatures increased from -59° to -48° C. The time lapse photography shows that clear ice accumulated steadily and faster than it could be melted away by the heating element. Another instance in hurricane Frances, 1961, revealed a continuous accumulation of clear ice over a 15-mi. course at a flight level temperature of -64° C. and with little or no horizontal temperature gradient in evidence. The temperature, recorded on a vortex thermometer, was verified by a rawinsonde report from San Juan, P.R., 80 mi. away. Thermometry on aircraft of any type presents many problems, especially in flight through cloud. The problem is magnified at higher air speeds. The NRL (axial flow) vortex

thermometer has been shown to have little dynamic correction or wet bulb effect at true air speeds less than about 450 kt. [6]. This thermometer is also subject to error under icing conditions; however, again the disturbance of flow through the vortex due to icing generally leads to dynamic heating and readings which are too high. The experience of the National Hurricane Research Project in flights through hurricanes shows no evidence of the large errors in vortex temperature readings at any level, and values obtained simultaneously from aircraft at different elevations have invariably been consistent thermodynamically within an error of 2° to 3° C. at most. It is not unreasonable, therefore, to assume that probable errors in temperature measurements mentioned above are reliable to within less than 5° C.

The evidence leaves little question as to whether aircraft icing occurs at air temperatures below -40° C. The unresolved question—one of considerable importance to cloud physics and in studies of convective processes—concerns the source of the icing. Clear icing on aircraft is most frequently observed in connection with flight through supercooled liquid water droplets. And most systematic investigations of aircraft icing have been conducted with propeller-driven aircraft in temperatures much higher than -40° C. At cruising speeds and operating altitudes of modern jet aircraft, there are at least four possible sources of icing at temperatures as low as -60° C. which should be examined:

1. Frozen raindrops may be melted by dynamic or frictional heating in the quasi-potential flow around the airfoil, then splash and freeze again as they strike the aircraft. This would require a restrictive class of drop sizes—large enough to move inertially through the boundary layer and strike the airframe, small enough to permit melting while approaching the airfoil or upon impact.

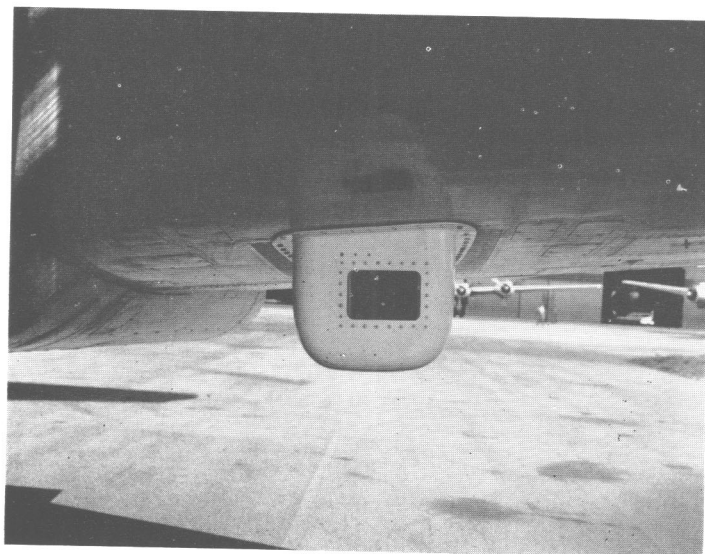


FIGURE 2.—Exterior mounting of the time-lapse camera on a W-57 aircraft. Ice which accumulated on the glass pane of the housing is shown in figure 1.

2. The freezing process in raindrops may begin with formation of a shell of ice at the boundary thus forming an "ice sphere" with a liquid core. This process has been studied in laboratory experiments by List [5] and his collaborators. If such a process operates in the free atmosphere (it remains to be shown that it does), the liquid cores of such partially frozen raindrops would tend to resist freezing since the air could not provide a direct sink for the latent heat of fusion released in the process. Thus raindrops enclosed in ice shells and carried rapidly upward by convective currents could remain at temperatures considerably higher than the temperature of the ambient air mass and provide a source of aircraft icing at air temperatures below -40°C . The computation of maximal temperature differences which could develop in this way presents many difficulties.

3. Large raindrops may be carried so rapidly upward by convective currents that they cannot cool to -40°C until they have reached a much colder atmospheric environment.

4. Finally, the spontaneous nucleation temperature may be pressure dependent or variable due to factors yet to be identified.

Of these four possibilities, this paper is concerned mainly with the third. The computations to be presented will show how much the cooling of raindrops may be expected to lag behind the change in environmental temperature as the drops are transported upward by convective currents.

2. COOLING OF RAINDROPS TRANSPORTED UPWARD BY VERTICAL CURRENTS

The transfer of heat between a liquid water droplet and its environment is ordinarily a diffusion process. Experience has shown this to be valid for small droplets in still air. For raindrops carried upward by strong vertical currents, the process is more complicated. In this case, the environmental air rushes by the raindrop at a relative speed of as much as 11 m. sec.⁻¹. If under these conditions the air serves in effect as an infinite heat sink, then the drop will cool by conduction. Conductive cooling of a spherical ventilated drop is proportional to the product of the spherical Laplacian of the temperature and the heat conductivity of water.

$$a^2 \nabla_s^2 T = \frac{\partial T}{\partial t} \quad (1)$$

where: $a^2 = k/(c\rho)$; k is heat conductivity of water; c is specific heat of water; ρ is density of water; T is temperature; and t is time.

Actually the drop is not spherical; moreover, the potential flow of air around the drop would result in at least two locations of inadequate ventilation where cooling would occur mainly by diffusion. Even with uniform ventilation the solution of (1) is an eigenvalue problem of

considerable complexity, especially in view of the fact that the temperature of the heat sink continually varies. However, it is clear that the rate at which raindrops cool by conduction is substantially greater than that which would result from the diffusion process. Since in nature the cooling occurs at some rate between that of conduction and that of diffusion, if one uses the diffusion equation to compute the difference in temperature between raindrops and air as the drops are swept upward, the result will *overestimate* the temperature difference that actually exists. Values obtained in this way will represent limiting extreme temperature differences which may exist between liquid water and its atmospheric environment.

The heat flux dQ/dt across the surface of a water drop due to diffusion is

$$\frac{dQ}{dt} = 4\pi r K (T_s - T_a) \quad (2)$$

where K is thermal conductivity of air; T_s is temperature of drop surface; T_a is temperature of air; and r is drop radius. This flux is related to drop cooling by

$$\frac{dQ}{dt} = -\frac{4}{3} \pi r^3 \rho c \frac{dT_a}{dt} \quad (3)$$

where T_a is drop mean temperature.*

Since

$$\frac{dT_a}{dt} = w_a \frac{\partial T_a}{\partial z}$$

where w_a is vertical speed of the drop, positive upward, then

$$\frac{\partial T_a}{\partial z} = -\frac{3K(T_s - T_a)}{\rho c r^2 w_a} \quad (4)$$

Vertical currents required for constant $(T_s - T_a)$ values.—Consider first the simple case in which the raindrop is carried upward at such a speed that the difference between its temperature and that of the environment remains constant, that is,

$$\frac{\partial}{\partial z} (T_s - T_a) = 0$$

If, as a first approximation, it is further assumed that the drop maintains its integrity and size, and is carried upward by an undiluted vertical air current, then

$$\frac{\partial T_a}{\partial z} = \Gamma_m$$

where Γ_m is the moist adiabatic lapse rate. Equation (4) may then be written

$$w_a = -\frac{3K(T_s - T_a)}{\rho c r^2 \Gamma_m} \quad (5)$$

*Note that while these computations distinguish between T_a and T_s , for most practical purposes they may be regarded as the same according to the findings of Gunn and Kinzer [3].

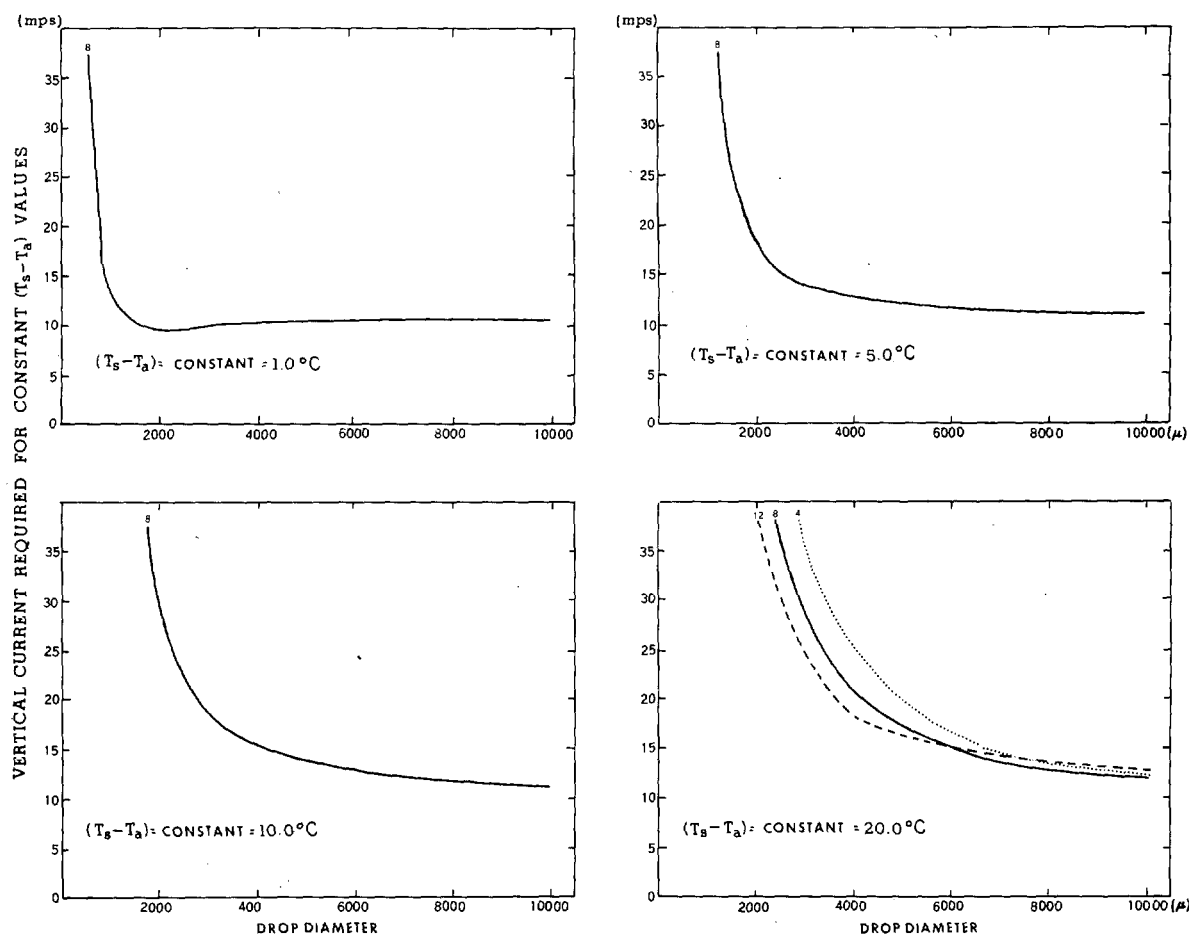


FIGURE 3.—Vertical air speed required to maintain a constant temperature difference between the air and the liquid water carried upward by the air.

and

$$w_a = -\frac{3K(T_s - T_a)}{\rho c r^2 \Gamma_m} + w_T \quad (6)$$

where w_T is terminal velocity, and w_a is vertical speed of the air current which is responsible for the temperature difference $(T_s - T_a)$.

We may now compute the speed of the transporting vertical current which would keep the temperature of the drop constantly higher than that of the air by some specified amount. Under the imposed conditions that

$$\frac{\partial r}{\partial z} = 0 \text{ and } \frac{\partial T_d}{\partial z} = \Gamma_m = \frac{\partial T_s}{\partial z}$$

figure 3 shows the results for $(T_s - T_a)$ values of 1° , 5° , 10° , and 20° C. with various drop sizes. w_T values were based upon the data of Gunn and Kinzer [3] adjusted for altitude. Computations were made for values of K and w_T at 8 km. (see fig. 4). Figure 5 shows the variation in w_T with height. Since variations of K with height are of opposite sign, these two tend to compensate one another. In figure 3, computations for $(T_s - T_a) = 20^\circ$ C. were

carried out at 4, 8, and 12 km. and are shown in the panel on the lower right. From these computations it is evident that the conservation of $(T_s - T_a)$ values, even of 1° C., would require vertical air currents with *mean speeds* in excess of 10 m. sec.⁻¹ for very large drops; and for small drops, e.g., 200μ , vertical currents of more than 200 m. sec.⁻¹ would be needed. For the drop to remain 20° C. warmer than the air at 12 km., a 2000μ drop would require a mean vertical air current in excess of 50 m. sec.⁻¹ It should be noted here that experience [7] has shown that drops larger than about 0.5 cm. (5000μ) tend to fracture into smaller drops before reaching terminal velocity.

Effect of evaporation and coalescence.—The computations so far have assumed no change in drop diameter with height. Actually, the diameter would continually increase as a result of coalescence and decrease from evaporation.

Figure 6 shows the net result of these two effects for a drop whose initial diameter is 2000μ . Here, we specify a cloud with 0.8 gm. m.⁻³ of liquid water comprised of 20μ droplets, and raindrops which rise at a rate which allows the drop to remain constantly 15° C. warmer than the air which carries it upward. The collection efficiency

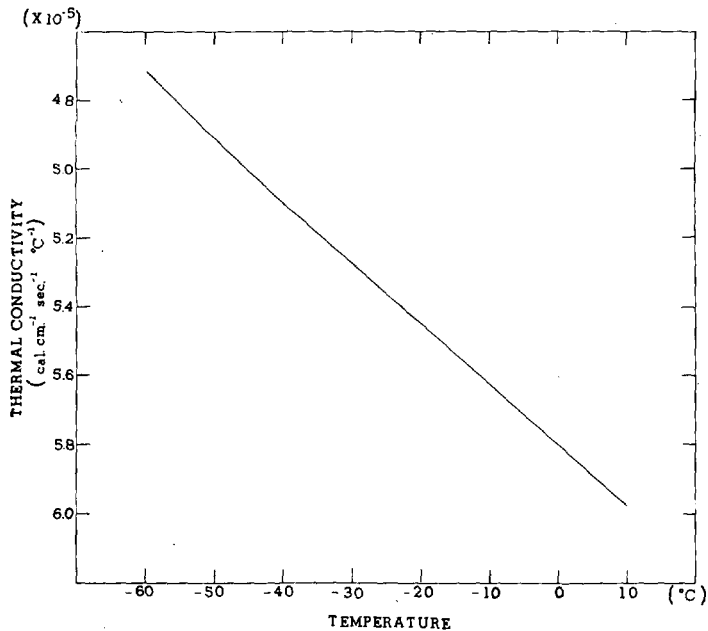


FIGURE 4.—Variation of K , the thermal conductivity of air, with temperature. Values from the Smithsonian Meteorological Tables have been extrapolated assuming variations to be proportional to the dynamic viscosity coefficient.

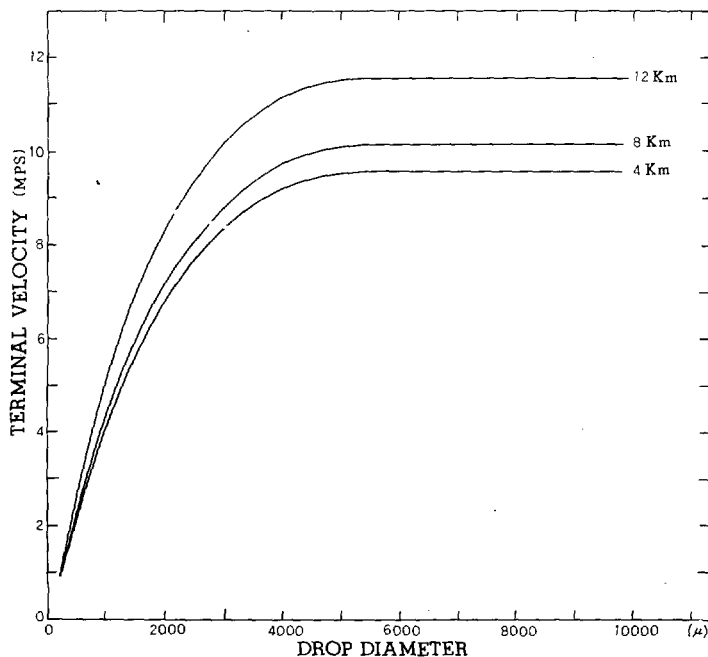


FIGURE 5.—Terminal velocity of raindrops at 4, 8, and 12 km. (After Gunn and Kinzer [3].)

values used are given by Johnson [4]. The reduction of drop size by evaporation is quite small compared to the increase by coalescence, a comparison which is accentuated with height.

Extreme differences between raindrop and air temperatures due to diffusive heat transfers.—Having first examined

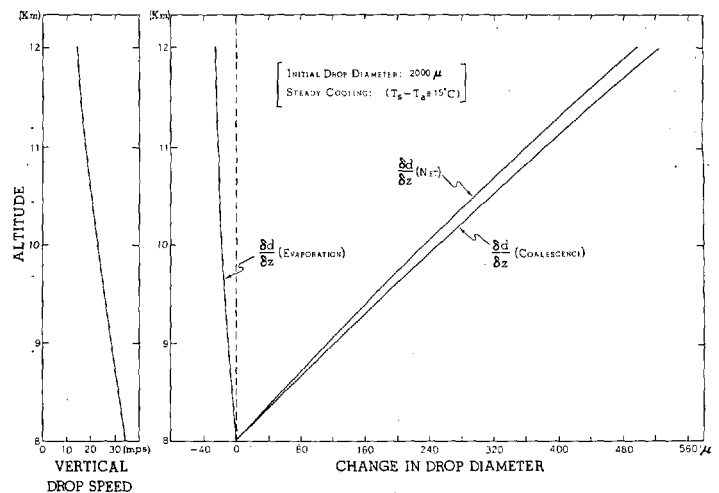


FIGURE 6.—Change in size of a raindrop due to coalescence and evaporation as it is transported upward in a vertical current. The initial drop size is 2000μ and the vertical current is defined as that which would keep the drop constantly 15°C . warmer than the air transporting it.

the mean vertical wind speed required to maintain a specified $(T_s - T_a)$ value, let us consider next the extreme temperature difference $(T_s - T_a)$ which could develop during the vertical wind speed of 20 m. sec.^{-1} , from near the 0°C . level to 12 km. , heat transfers being accomplished by diffusion. Here, we no longer impose a requirement for constant air-water temperature difference and allow the droplet to grow by coalescence. For this computation, equation (4) may be written

$$\int_{z_1}^{z_2} dT_d = -\frac{3}{\rho c r^2 w_d} \int_{z_1}^{z_2} K(T_s - T_a) dz \quad (7)$$

In the integration of (7), the following initial conditions and definitions were used:

$$(T_s - T_a) = 1^\circ\text{C.}$$

$$T_s = T_d$$

$$\left(\frac{\partial T_d}{\partial z}\right)_0 = \Gamma_m$$

$$T_a(z_1) = T_a(z_0) + \Delta z \Gamma_m$$

$$T_s(z_1) = T_s(z_0) + \Delta z \left(\frac{\partial T_d}{\partial z}\right)_0$$

The increase in drop size, where the subscript (0) refers to initial values, is due to coalescence and the computation of r and w_d assumed a uniform environment with liquid water content 0.8 gm. m.^{-3} consisting of cloud droplets 20μ in diameter, and a collection efficiency following values given by Johnson [4]. The integration was carried out for increments of 0.25 km. The net temperature change of the droplet considered that the temperature of coalescing 20μ droplets was that of the ambient air.

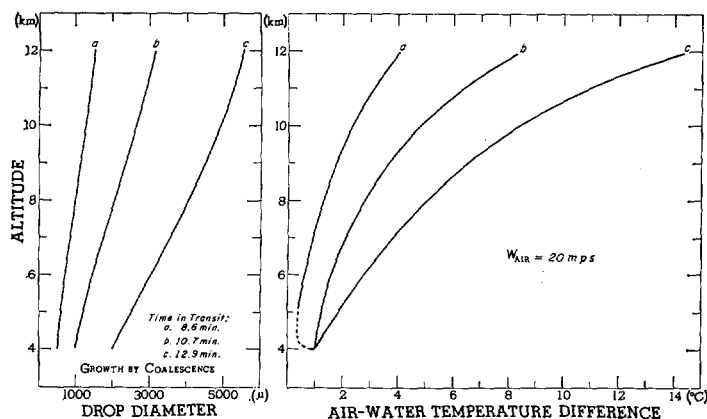


FIGURE 7.—Difference in temperature of large raindrops and the ambient air as the raindrops are carried upward from 4 to 12 km. by a deep-layer steady-state updraft of 20 m. sec.⁻¹. This computation assumes: (1) drop size growth to coalescence from 20 μ cloud droplets; and (2) all heat transfers occur through diffusion.

The results of these computations are shown in figure 7. Under the influence of a steady-state deep-layer updraft of 20 m. sec.⁻¹, drops having an initial size of 500 μ would rise from the 4-km. level to 12 km. in 8.6 min., would grow by coalescence to a size of 1514 μ , and would reach the 12-km. level 4.0° C. warmer than the ambient air, if cooling were due solely to diffusion. A drop whose initial size was 2000 μ would require 12.9 min. to reach the 12-km. level, would increase in size to 5520 μ and, if it did not fracture during ascent, would arrive there 14.3° C. warmer than the ambient air.

Discussion.—In quiet cloud-free air raindrops half a centimeter in diameter can sometimes fall without fracture. In rising through a deep layer of convective cloud, it is questionable whether such drops, subject to turbulent processes, could travel upward an appreciable fraction of the distance needed to produce or maintain an air-water temperature difference of 20° C. without fracturing into two or more smaller drops in which the temperature difference would quickly decrease.

Experience has shown that vertical gusts of 20 m. sec.⁻¹ are not uncommon in squall lines [10] and there is some evidence of extreme gusts in excess of 70 m. sec.⁻¹. In some sectors of hurricanes there is evidence [9] of large vertical mass transports in convective chimneys near the eye, estimated to be 10 m. sec.⁻¹ or more. However, it is generally conceded by cloud physicists that the active life of any one convective cell is substantially less than half an hour. Unless there exists characteristically, and more generally, the kind of steady-state deep updrafts which Fujita and Byers [2] have associated with hail clouds, then it would appear unlikely that vertical motions in ordinary large cumuli or in hurricanes would attain the intensity and persistence necessary to account

for large differences in temperature between waterdrops and the ambient air.

Finally, it must be recalled that the computations of raindrop cooling have considered that all heat transfers occur by diffusion. Since this substantially *underestimates* the rate of cooling over the ventilated portion of the drop surface, obviously a raindrop cannot acquire a temperature 20° C. lower than the temperature of its environment without even more rapid vertical transport than is indicated by these computations.

While these computations do not provide as exact a solution as might be desired for some purposes, they do furnish useful information about the limiting conditions which must exist if there is to be significant difference in temperature between raindrops and the air environment. From what is presently known of convective processes, it is very doubtful that such conditions exist as frequently as icing conditions are encountered in air much colder than -40° C., the spontaneous nucleation temperature.

3. SUMMARY

There is little question that aircraft icing occurs frequently during flight through convective clouds with air temperature -60° C. or lower. The important question is whether the icing occurs in the presence of liquid water which is also colder than the spontaneous nucleation temperature (-40° C.). The computations here attempt to determine whether raindrops carried rapidly upward by convective currents could arrive at cloud top levels as much as 20° C. warmer than the air around them. To accomplish this, raindrops initially larger than 2000 μ in diameter would have to come under the influence of a deep-layer steady-state updraft of more than 20 m. sec.⁻¹ for a period of about 15 min., and during this period grow to the size of more than 5500 μ without fracturing. All factors considered, it is unlikely that such large air-water temperature differences may develop through this process unless intense, deep-layer steady-state updrafts are more commonplace than present experience indicates. Yet to be investigated is the possibility that the icing may be due to the impact of partially frozen droplets with liquid cores, the rate of compressional and frictional heating of droplets approaching the airfoil, and the possibility that the spontaneous nucleation temperature may be pressure dependent or variable due to other factors.

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